Our conclusions can be summarized in the following points:

a) The present data on $p\bar{p}$ high momentum transfer elastic scattering can be described by the Orear formula (1).

b) These data are in disagreement with the predictions of the Fermy type statistical models, and seem to support the models of high momentum transfer processes like the Wu and Yang model or Hagedorn thermodynamical model.

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SUM RULES FOR STRONG INTERACTIONS *

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Sum rules for strong interactions are derived from analyticity and high energy bounds for the scattering amplitudes of particles endowed with spin. Those bounds, much more stringent than for the spinless case, are strongly suggested by unitarity.

In a recent paper [1] general consequences from the algebra of current components have been deduced. In particular, it has been shown that, using the equal time commutation relations of the time components of currents,

$$[j_0^{(\alpha)}(x), j_0^{(\beta)}(x')]_{t=t'} = C_\gamma^{\alpha\beta} j_0^{(\gamma)}(x)\,\delta(\boldsymbol{x}-\boldsymbol{x}') \ , \quad (1)$$

one obtains a relation of the form

$$\frac{1}{\pi}\int a(\nu, u_1, u_2, t)\mathrm{d}\nu = F(t) \ , \quad (2)$$

where a is defined through the expansion of the amplitude

$$t_{\mu\nu} = \tfrac{1}{2}\int \mathrm{d}^4x \exp(\mathrm{i}q_2x)\langle p_2|[j_\mu^{(\alpha)}(x), j_\nu^{(\beta)}(0)]|p_1\rangle \ , \quad (3)$$

as

$$t_{\mu\nu} = aP_\mu P_\nu + b_1 P_\mu Q_\nu + b_2 P_\mu \Delta_\nu + \dots \ , \quad (4)$$

where $P = \frac{1}{2}(p_1+p_2)$, $Q = \frac{1}{2}(q_1+q_2)$, $\Delta = p_2 - p_1$; $p_1+q_1 = p_2+q_2$; $u_1 = q_1^2$, $u_2 = q_2^2$; $\nu = (PQ)$, $t = \Delta^2$.

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and, for simplicity, $|p_1\rangle$, $|p_2\rangle$ are taken as spinless particles.

As already pointed out in refs. 1 and 2, eq. (2) exhibits the very peculiar and important feature that the dependence on the "masses" u_1 and u_2 associated with the external currents is washed out by the ν integration.

The independence of the right hand side of eq. (2) on u_1 and u_2 can be also expressed by saying that the residues of all singularities of the left hand side in u_1 and u_2 compensate through the ν integration.

Thus a first consequence can be drawn by multiplying both sides of (2) by $(u_1 - m_1^2)(u_2 - m_2^2)$ and performing the limits $u_{1,2} \to m_{1,2}^2$. m_1 and m_2 are the physical masses of strongly interacting particles with the same quantum numbers of the currents $j_\mu^{(\alpha)}$, $j_\nu^{(\beta)}$ (e.g. the ρ-meson if j_μ is the isospin current). In this limit a is dominated by the graph of fig. 1 and we write the rigorous relation

$$\lim_{u_{1,2}\to m_{1,2}^2} (u_1 - m_1^2)(u_2 - m_2^2)a(\nu, u_1, u_2, t) = = (\text{const.}) \operatorname{Im} A(\nu, t) , \quad (5)$$

where $A(\nu,t)$ is defined through the following decomposition of the ρ-π scattering amplitude:

$$T = (\epsilon_1 P)(\epsilon_2 P)A + \tfrac{1}{2}\{(\epsilon_1 P)(\epsilon_2 Q) + (\epsilon_2 P)(\epsilon_1 Q)\}B + + (\epsilon_1 Q)(\epsilon_2 Q)C_1 + (\epsilon_1\epsilon_2)C_2 . \quad (6)$$

In so doing, one derives from eq. (2) the sum rule

$$\int \operatorname{Im} A(\nu, t)\, d\nu = 0 , \quad (7)$$

which involves only the scattering amplitude of strongly interacting particles.

We want to stress the fact that eq. (7) is actually independent of any detailed assumption on the current-current commutators. Indeed it could be obtained from the commutation relations between any couple of vector "currents" provided this commutator involves (because of locality) the presence of a $\delta(x-y)$ or of its higher finite order derivatives.

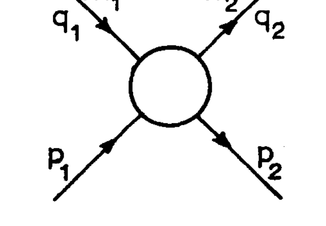

Fig. 1.

This suggests that the "strong interaction sum rule" (7) might be deduced in a more direct way only from strong interaction requirements like analyticity, unitarity and high energy limits *;

The aim of this work is to show that this is indeed the case and that eq. (7) is a very particular case of a general family of strong interaction sum rules involving particles with higher spin.

Our discussion of the "strong interaction sum rules" stems from the fact that if an analytic function $f(\nu)$ satisfying a dispersion relation

$$f(\nu) = \frac{1}{\pi} \frac{\operatorname{Im} f(\nu')}{\nu' - \nu} d\nu' \quad (8)$$

is subject to the asymptotic bound, for $\nu \to \infty$,

$$|f(\nu)| < \nu^\beta, \quad \beta < -1 , \quad (9)$$

it must satisfy the condition

$$\int \operatorname{Im} f(\nu)\, d\nu = 0 .$$

The physical importance of the previous formulae follows from the fact that in the case of scattering of higher spin particles some of the amplitudes are forced by unitarity to be subjected to extremely stringent high energy bounds, like those in (9) and therefore satisfy eqs. like (10) **.

We will fund thus that, for higher spins, unitarity leads for some amplitudes to bounds much stronger than $s(\ln s)^2$ obtained by Froissart [3] for the spinless case. This is due to the appearance in the unitarity sum over intermediate states of additional energy powers coming from the projection operators of higher spin particles.

Let us start with a very heuristic illustration of the Froissart bound for a scalar particle amplitude $A(s,t)$. Let us use at high energy the optical theorem together with the obvious condition that the total cross section is larger than the elastic one,

$$\operatorname{Im} A(s,0) > \frac{1}{16\pi\sqrt{s(s-4m^2)}} \int_{-s+4m^2}^{0} |A(s,t)|^2\, dt . \quad (11)$$

If we assume a constant shape of the diffraction peak, we get

$$|A(s,0)|^2/s < \text{const.} \operatorname{Im} A , \quad (12)$$

leading to the simple condition

* One of the authors (S.F.) wishes to thank M.Gell-Mann for a very illuminating advice on this point.

** The close connection between sum rules and high energy limits has been previously noted by S.Gasiorowicz (to be published).

$$|A(s,0)| < \text{const. } s \ . \qquad (13)$$

Of course the crude condition of constant diffraction peak could be dropped by allowing slow logarithmic variation (e.g. like the case of moving Regge poles); this would involve extra logarithmic terms in the high energy bound. Let us now apply the same kind of argument to ρ-π scattering where we use the "orthogonal decomposition"

$$T = \alpha I_\alpha + \beta I_\beta + \gamma I_\gamma + \delta I_\delta \ , \qquad (14)$$

where

$$I_\alpha = (\epsilon_1 P')(\epsilon_2 P'), \quad I_\beta = \tfrac{1}{2}\{(\epsilon_1 P')(\epsilon_2 Q) + (\epsilon_2 P')(\epsilon_1 Q)\},$$

$$I_\gamma = (\epsilon_1 Q)(\epsilon_2 Q), \quad I_\delta = (\epsilon_1 N)(\epsilon_2 N),$$

$$P'_\mu = P_\mu - \frac{(PQ)}{Q^2} Q_\mu \ , \quad N_\mu = \epsilon_{\mu\nu\rho\sigma} P_\nu Q_\rho \Delta_\sigma \ ,$$

$$(P'Q) = (P'N) = (QN) = 0 \ , \qquad (15)$$

and the amplitudes α, β, γ and δ are connected to A, B, $C_{1,2}$ by linear combinations.

Using again the optical theorem and asking that the total cross section be larger than the α, β, γ, δ contributions † we get ($P'^2 \sim s^2$ for $s \to \infty$)

$$\begin{aligned} s^4 \int |\alpha(s,t)|^2 \, dt &< \text{const.} s^2 \sigma^{tot} \ , \\ s^2 \int Q^2 |\beta(s,t)|^2 t \, dt &< \text{const.} s^2 \sigma^{tot} \ , \\ \int Q^2 |\gamma(s,t)|^2 t^2 \, dt &< \text{const.} s^2 \sigma^{tot} \ , \\ s^4 \int Q^4 |\delta(s,t)|^2 t^4 \, dt &< \text{const.} s^2 \sigma^{tot} \ . \end{aligned} \qquad (16)$$

Finally, expressing α, β, γ and δ in terms of A, B, $C_{1,2}$ and using the constant shape assumption, we obtain the bounds

$$|A(s,0)| < \text{const. } s^{-1}, \quad |B(s,0)| < \text{const.}, \text{ and}$$
$$|C_{1,2}(s,0)| < \text{const. } s \qquad (17)$$

Of course our "derivation" of the bounds (17) is very heuristic. It is likely that, as in the spinless case, the more systematic use of unitarity *in each partial wave* might lead to rigorous bounds which, apart from logarithmic factors, coincide with our eqs. (17).

If we are in presence of isospin it is reasonable to think that more stringent limitations could follow for the isospin flip amplitudes. We shall assume that, if the behaviour of an isospin amplitude for scalar particles is s^α, the corresponding behaviours for A, B, C_1 and C_2 are given by

† Note that no interference terms appear because of our orthogonal decomposition.

$$A(s,0) \sim s^{\alpha-2} \ , \quad B(s,0) \sim s^{\alpha-1} \ , \quad C_{1,2}(s,0) \sim s^\alpha . \qquad (18)$$

Those asymptotic formulae could be obtained from a Regge pole model, since the exchange of spin α leads to high energy behaviours $s^{\alpha-2}$, $s^{\alpha-1}$, s^α for A, B, $C_{1,2}$ respectively. The conclusion of this discussion is that in ρ-π scattering *we get a sum rule of the kind* (10) *for A if* $\alpha < 1$, *and for B if* $\alpha < 0$ *.

It is important to realize that the results obtained are in no way limited to spin one particles. For example in π-N scattering, writing $T = A + (\gamma Q)B$, we get $A \sim s^\alpha$, $B \sim s^{\alpha-1}$, whereas in N-N scattering the amplitudes T_i defined through [4]

$$T = \Sigma_i T_i P_i \ ,$$

$$P_1 = 1^P 1^N, \quad P_2 = i[(\gamma^N P)1^P + (\gamma^P N)1^N] \ ,$$

$$P_3 = (i\gamma^N P)(i\gamma^P N), \quad P_4 = (\gamma^N \gamma^P), \quad P_5 = \gamma_5^N \gamma_5^P$$

show the behaviour s^α, $s^{\alpha-1}$, $s^{\alpha-2}$, $s^{\alpha-1}$, s^α respectively, where α depends upon the isospin decomposition in the crossed (N-$\bar{\text{N}}$) channel.

A general investigation of the meaning and experimental validity of the strong interaction sum rules is deferred to further work. We treat here the simple forward ρ-π scattering already considered in ref. 2. We treat the isospin variable be decomposing the amplitude T as $T = T_0 P_0 + T_1 P_1 + T_2 P_2$, where $P_{0,1,2}$ are the projection operators in the isospin eigenstates of the crossed channel $\pi + \pi \to \rho + \rho$. The high energy behaviour of T_1 will be dominated by the ρ trajectory (experimentally $\alpha_\rho(0) \approx 0.5$), whereas T_2 will be dominated by double charge exchange. We shall here assume $\alpha_{++}(0) < 0$.

Our discussion means that we shall have the two sum rules **

$$\int \text{Im } A^{(1)}(\nu,0)\, d\nu = 0 \ , \qquad (19)$$

$$\int \text{Im } B^{(2)}(\nu,0)\, d\nu = 0 \ . \qquad (20)$$

If we keep only π, ω, φ as intermediate particles, we obtain ***

* The idea that strong interaction sum rules can be obtained from asymptotic limits has been independently developed by L. D. Soloviev in a beautiful investigation (Dubna preprint). However his high energy assumptions differ from ours. For example for π-N scattering the assumption $|B| < s^{-1} (\ln s)^{-a}$, $a > 1$, is stronger than what implied by our unitarity arguments.

** Of course $\int \text{Im } A^{(2)}(\nu,0)\, d\nu = 0$ is trivially satisfied by crossing symmetry.

*** The following couplings have been used: $g_{\rho\pi\pi} \epsilon_{ijk} \rho^i_\mu \pi^j \partial_\mu \pi^k$, $g_{\omega\rho\pi} \epsilon_{\alpha\beta\gamma\delta} \partial_\alpha \omega_\beta \partial_\gamma \rho^i_\delta \pi^i$, $g_{\varphi\rho\pi} \epsilon_{\alpha\beta\gamma\delta} \partial_\alpha \varphi_\beta \partial_\gamma \rho^i_\delta \pi^i$.

$$(g^2_{\omega\rho\pi} + g^2_{\varphi\rho\pi})m^2_\rho - 4g^2_{\rho\pi\pi} = 0 \ , \qquad (21)$$

$$(\nu_\omega + m^2_\rho)g^2_{\omega\rho\pi} + (\nu_\varphi + m^2_\rho)g^2_{\varphi\rho\pi} - 4g^2_{\rho\pi\pi} = 0 \ , \qquad (22)$$

with $\nu_{\omega,\varphi} = \frac{1}{2}(m^2_{\omega,\varphi} - m^2_\rho - m^2_\pi)$.

Subtracting (22) from (23),

$$\nu_\omega g^2_{\omega\rho\pi} + \nu_\varphi g^2_{\varphi\rho\pi} = 0 \ . \qquad (23)$$

Since ν_ω is practically zero, (23) tells that the ratio $g_{\varphi\rho\pi}/g_{\omega\rho\pi}$ is very small, in good agreement with the experiment. In the same way we get, as in ref. 2, as in [2], a reasonable relation between $g_{\rho\pi\pi}$ and $g_{\rho\omega\pi}$.

In conclusion we have derived sum rules connecting only strong interaction quantities as a consequence of analyticity and reasonable arguments about high energy behaviour. It is important to emphasize the fundamental role played by spin in this sort of arguments: we have no relation for spinless particles, and the number of relations increases very fast with the value of the spins of the particles involved. On the other side the number of free coupling constants increases with spin more or less with similar rate as the number of equations. We believe that our strong interaction sum rules will play an important role in the development of elementary particle physics. The sum rules are probably the relativistic generalization of the so-called bootstrap conditions which could be obtained by imposing analogous unitarity bounds on each partial wave [5].

A beautiful new feature of our results is the connection between constants of different dimensionality like the "electric" coupling $g_{\rho\pi\pi}$ and the "magnetic" coupling $g_{\rho\omega\pi}$. It is hoped that this will be useful to obtain new interconnections between elementary particle properties which are unattainable by purely group theoretical methods.

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EVIDENCE FOR A BOSON RESONANCE AT 1.63 GeV IN π^-p INTERACTIONS AT THE INCIDENT MOMENTUM OF π^--MESONS 4.7 GeV/c

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A new resonance in the $\pi^+\pi^-\pi^-$ system produced in the reaction $\pi^-p \to p\pi^+\pi^-\pi^-$ is found in 55 cm hydrogen bubble chamber photographs, with π^--momentum 4700 ± 30 MeV. The mass value is 1.63 ± 0.03 GeV, $\Gamma \approx 0.1$ GeV, isotopic spin $I \geq 1$, production cross-section ~ 0.1 mb.

When analysing photos, obtained in a 55 cm hydrogen bubble chamber exposed in the beam of π^--mesons at 4700 ± 30 MeV we found a new resonance in $\pi^+\pi^-\pi^-$ system, produced in a reaction:

$$\pi^- + p \to \pi^- + p + \pi^+ + \pi^- \qquad (1)$$

The mass of the new resonance is 1.63 ± 0.03 GeV, $\Gamma \approx 0.1$ GeV, isospin $I \geq 1$, and the cross section of its production in reaction (1) is ~ 0.1 mb.

Four-prong stars were studied in 60 000 photos and 504 events of reaction (1) were selected. The event was accepted when for this particular hypothesis $\chi^2 \leq 10.5$. Protons and π^+-mesons were identified visually up to momentum 1 GeV/c and by ionization measurement for momentum range up to 1.8 GeV/c. Events with higher momentum of positive particles if not determined in a unique way were rejected (7.5% of selected sample of reaction (1).

On fig. 1 a spectrum of effective masses of